

ANALYSIS OF STRESSES CAUSING LAND SUBSIDENCE

By BEN E. LOFGREN, Sacramento, Calif.

Abstract.—Land subsidence in areas of intensive ground-water withdrawal is attributed to the compaction of deposits caused by increased effective loading stresses. As demonstrated at the Pixley recorder site, subsidence is related directly to effective-stress changes computed from water-level data. Effective-stress changes are sometimes difficult to calculate in a complex of aquifer systems, even with ample water-level data. Effective stresses are changed in two principal ways: (1) Water-table fluctuations change the buoyant support of grains in the zone of the change, and (2) a change of the water table or of the piezometric head, or both, may induce hydraulic gradients and seepage stresses in the deposits. These stress changes are additive in their effect, and together cause compaction.

Nowhere have the activities of man produced more extensive land subsidence than in the San Joaquin Valley of central California (fig. 1). Here, the accelerated pumping of ground water for irrigation, especially since 1940, has caused more than 3,000 square miles of land to subside. In the area of maximum subsidence 10 miles southwest of Mendota, a water-level decline of 300 feet has resulted in 26 feet (1966) of settlement since 1930. Wherever studied in the San Joaquin Valley, a correlation exists between subsidence and water-level decline. The magnitude of the subsidence is dependent on the effective-stress increase, the compressibility of the deposits, the thickness of the compressible beds, the time the increased stress has been applied, and possibly the rate and type of stress applied. It is also dependent on the past stress history—whether the increased stress is being applied for the first time or has been attained or exceeded previously.

Depending on the nature of the deposits, compaction may be (1) largely elastic, in which case stress and strain are proportional, independent of time, and reversible, or (2) principally nonelastic, resulting from a rearrangement of the granular structure in such a way that the volume of the deposits is permanently decreased. In general, if the deposits are coarse sand and gravel, the compaction will be small and chiefly

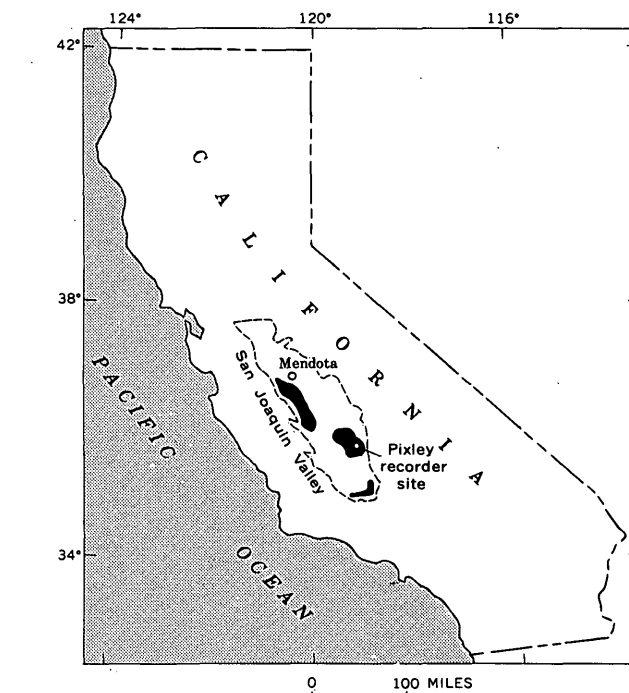


FIGURE 1.—Principal subsidence areas (shaded) in the San Joaquin Valley, and location of the Pixley recorder site.

elastic and reversible, whereas if they contain fine-grained clayey beds, the compaction will be much greater and chiefly inelastic and permanent. In either case, a one-directional compression of the deposits occurs which results in a subsidence of the land surface.

Figure 2 shows the correlation that exists in the confined aquifer system near Pixley (fig. 1) between water-level change and compaction of the unconsolidated deposits. At this location, both the confined water level in well 23/25-16N3 and the recorded compaction in well 23/25-16N1 show an immediate response to pumping effects of nearby irrigation wells. The compaction recorder (for description, see Lofgren, 1961) is anchored at a depth of 760 feet and measures the

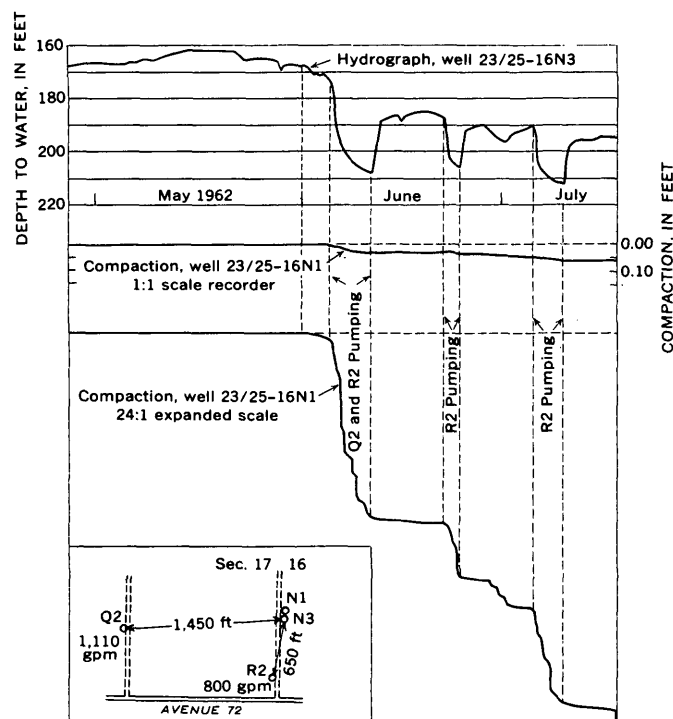


FIGURE 2.—Water-level change and compaction near Pixley.

vertical shortening that occurs in the formation, principally in the depth interval from 355 to 760 feet. As shown in the lower graph (24:1 expanded scale), compaction of the aquifer system begins immediately after the water level in nearby observation well 23/25-16N3 begins to fall, and compaction virtually stops soon after the water level in the observation well starts to recover.

Figure 3 shows the interrelationship between compaction, land subsidence, water-level fluctuations in the semiconfined (hydrograph, 23/25-16N4) and confined (hydrograph, 23/25-16N3) aquifer systems, and change in effective stress at this same recorder site near Pixley for 6 years of record. Subsidence of bench mark Q945 is determined by U.S. Coast and Geodetic Survey levelings from distant and stable bench marks. During this period measured compaction to a depth of 760 feet was 75 percent of the total subsidence. Thus, 25 percent of the vertical shortening was due to compaction below 760 feet.

Comparison of the compaction rate with fluctuations in water level in the semiconfined and confined aquifer systems indicates that compaction began each year during the period of rapid head decline in the aquifer system, continued through the pumping season, and ceased during the early stages of head recovery. Although the correlation between compaction and artesian-head change in the confined aquifer system is

good, the correlation with changes in effective stress in the confined aquifer system (center graph, fig. 3) is closer. Of special interest in relating compaction to changes in effective stress is the position of straight line $X-X'$. Most of the compaction occurred when the effective-stress curve was below line $X-X'$ (that is, when the stress was higher), whereas little or no compaction occurred when the effective stress was above line $X-X'$. Compaction continued for several weeks each year after the date of maximum effective stress and, in all instances, terminated before line $X-X'$ was reached during recovery. In general, the amount of compaction in any year is related to the area below line $X-X'$ bounded by the effective-stress curve.

TYPES OF STRESSES

Three different types of stresses are involved in the compaction of an aquifer system. These are closely interrelated, yet of such different nature that a clear distinction is of utmost importance. The first of these is a gravitational stress, caused by the effective weight of overlying deposits, which is transmitted downward through the grain-to-grain contacts in the deposits. The second, a hydrostatic stress due to the weight of the interstitial water, is transmitted downward through the water. The third is a dynamic seepage stress exerted on the grains by the viscous drag of vertically moving interstitial water. The first and third are additive in their effect and together comprise the grain-to-grain stress which effectively changes the void ratio and mechanical properties of the deposit; it is commonly known as the "effective stress." The second type of stress, although it tends to compress each individual grain, has virtually no tendency to change the void ratio of the deposit and is referred to as a "neutral stress."

Of the various methods used in analyzing the effect of these stresses in a compacting aquifer system (Taylor, 1948, p. 203), only two are considered here. Although they vary in their conceptual approach, these methods give the same mathematical results and can be used to check one another. The classical method, the approach most often used in practical soil-mechanics problems, considers the geostatic load, or combined total weight of grains and water in the system, and the neutral, or hydrostatic, stress. The second method considers the static gravitational stress of the grains, which comprises their true weight above the water table and submerged (buoyed) weight below the water table, and the vertical seepage stresses that may exist in the system. Inasmuch as changes in the effective grain-to-grain stress (both gravitational and stress due to seepage) are directly responsible for the com-

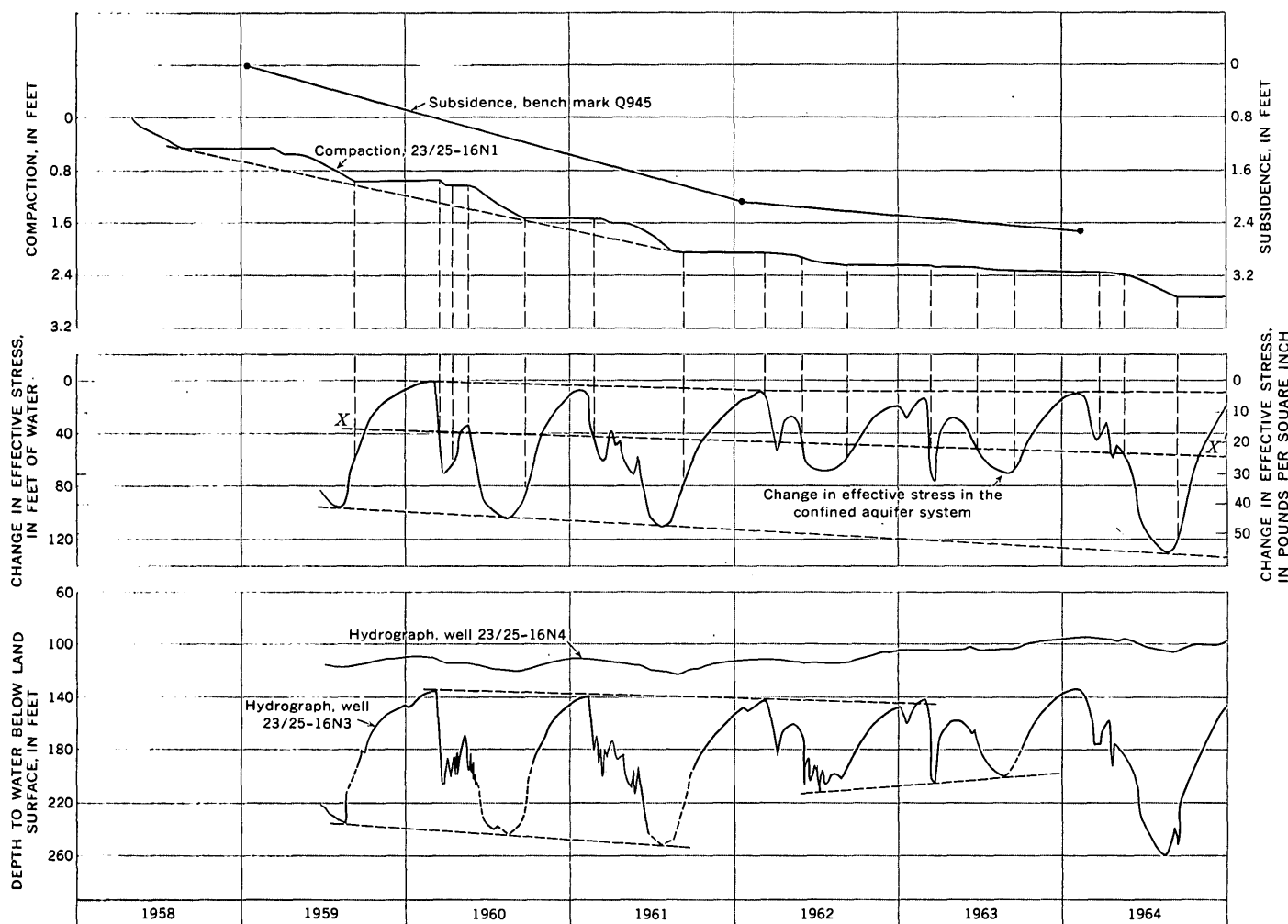


FIGURE 3.—Land subsidence, compaction, water-level fluctuations, and change in effective stress, 3 miles south of Pixley.

paction of the deposits and are directly related to changes in head in an aquifer system, this second approach has proved the simplest and clearest in our subsidence investigation.

The Terzaghi theory of consolidation (Terzaghi and Peck, 1948, p. 233) commonly is used to estimate the magnitude and rate of compaction that will occur in fine-grained clayey deposits under a given change in stress. According to the theory, compaction results from the slow escape of pore water from the stressed deposits, accompanied by a gradual transfer of stress from the pore water to the granular structure of the deposits. When an increment of loading stress is applied to an aquitard (fine-grained interbed), the entire stress increase first is borne by the interstitial water because no increase in grain-to-grain stress can occur until drainage from the aquitard begins. A high hydraulic gradient at the surface of the aquitard causes rapid drainage from the pores near that surface. Then, as the excess pressure gradually decreases, the intergranular stress increases and the void ratios

decrease; this slow process always is in a more advanced state near the surfaces of the aquitard and at a less advanced state near its middle.

Effective stress increases and compaction occurs in a water-bearing bed only as rapidly as water can move out. A slowly draining aquitard may take weeks or years to adjust to an applied stress increase, whereas a coarse-grained aquifer will adjust quickly. In either case, one-dimensional consolidation results, which is directly related to the change in effective grain-to-grain stress.

RELATION OF CHANGE IN EFFECTIVE STRESS TO WATER-LEVEL FLUCTUATIONS

Water-level fluctuations in subsidence areas change effective stresses in the following two ways:

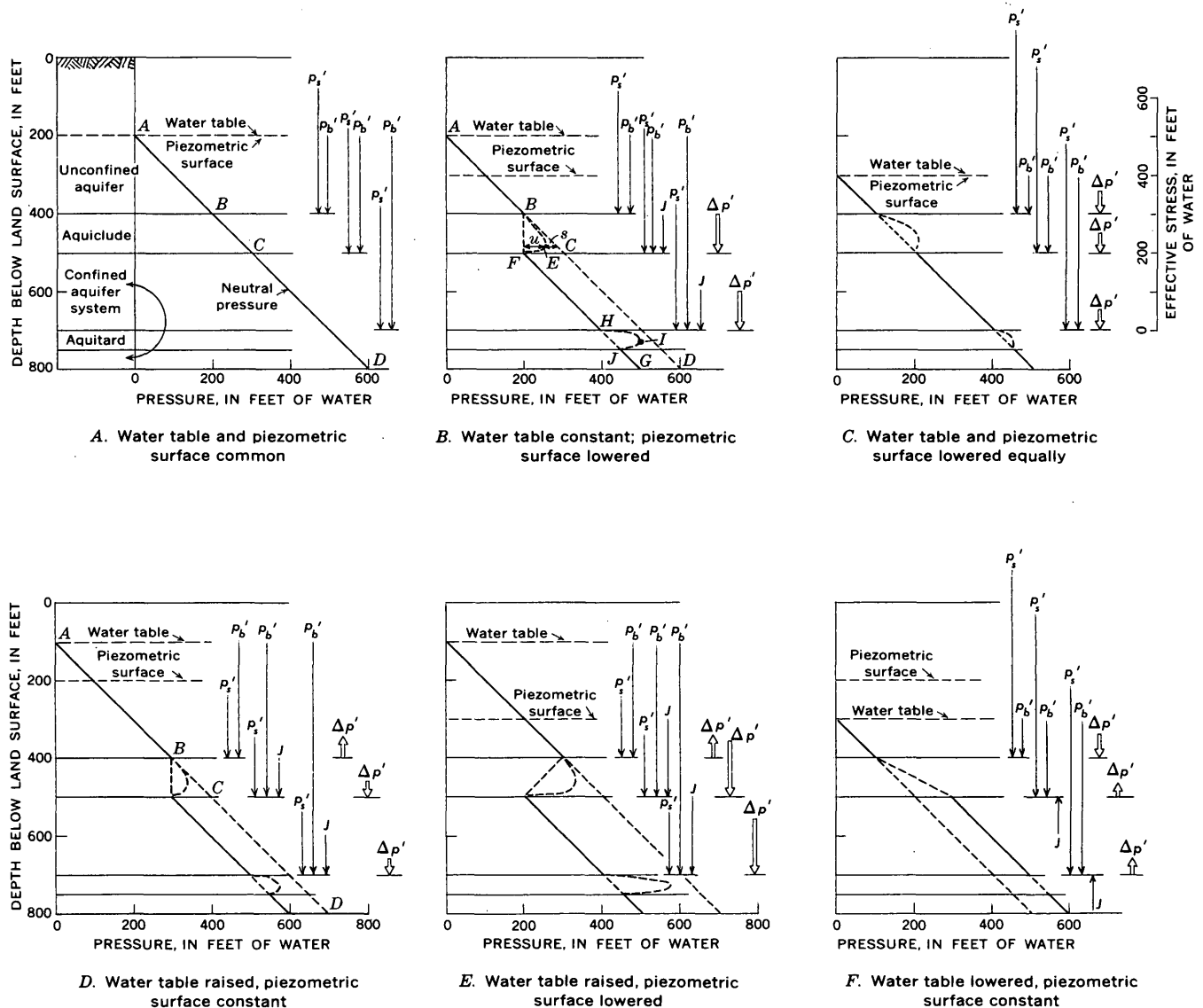
1. A rise of the water table increases the buoyant support of grains in the zone of the change, and a decline decreases the buoyant support; these changes in gravitational stress are transmitted downward to all underlying deposits.

2. A change in position of the water table or the piezometric surface (artesian head), or both, may induce vertical hydraulic gradients across confining or semiconfining beds and thereby produce a seepage stress; this stress is algebraically additive to the gravitational stress that is transmitted downward to all underlying deposits. A change in effective stress also results if preexisting seepage stresses are altered in direction or magnitude by a change in head.

In the following analysis of stresses, consideration will be given to compaction under two different hy-

draulic regimes, namely (1) water-table (unconfined) conditions in which no vertical hydraulic gradients exist, and (2) artesian (confined) conditions, in which vertical gradients exist and result in seepage stresses that usually represent the major part of the change in effective stress. Generally, changes in effective stress involve changes in both buoyant support and in seepage forces, with both time and depth.

Figure 4 shows the components of effective stress at three depths in an assumed hydrologic system that consists of a confined aquifer system having an interbedded aquitard and an overlying unconfined aquifer.



All stresses in feet of water; based on assumed porosity=0.40 and specific gravity=2.70

p'_s = effective stress due to weight of unsaturated deposits

p'_b = effective stress due to buoyant weight of submerged deposits

J = seepage stress

$\Delta p'$ = change in total effective stress from condition A

FIGURE 4.—Effective-stress diagrams for a confined aquifer system overlain by an unconfined aquifer.

For purposes of demonstration, a porosity, n , of 40 percent and an average specific gravity, G , of 2.7 for the individual mineral grains are assumed. All stresses in the system are expressed in units of feet of water (1 foot of water equals 0.43 pounds per square inch) so that they are directly additive and are relatable to changes in hydraulic head. Thus, the unit weight of deposits above the water table (ignoring the contained moisture), γ_a , may be expressed as

$$\gamma_a = (1-n) \cdot G \cdot \gamma_w,$$

in which γ_w is the unit weight of water.

Correspondingly, the buoyant (submerged) unit weight, γ' , of the deposits below the water table equals

$$\gamma' = (1-n)(G-1)\gamma_w$$

Using the above assumed numerical values for n and G and expressing γ_w as 1 foot of water, the effective stress due to the weight of deposits above the water table equals about 1.6 feet of water per foot of thickness, and the effective stress due to the weight of deposits below the water table equals approximately 1.0 foot of water per foot of thickness.

UNCONFINED (WATER-TABLE) CONDITIONS

Before pumping lowers the water table, in an unconfined aquifer, the deposits are generally in equilibrium with the natural effective stresses. For any element of the deposit, these effective stresses equal the true weight (per unit area) of the unsaturated deposits and the buoyant weight (per unit area) of the saturated deposits that overlie the element. As the water table is lowered, the effective unit weight of the deposits in the dewatered zone increases from the submerged unit weight, γ' , (weight of solids minus the weight of the displaced water) to the unit weight, γ_a , of the solids. Thus, the effective grain-to-grain stress in the aquifer below the dewatered zone is increased by the loss of buoyant support in the dewatered zone and the increase in effective stress at any depth below the lowered water table is equal to the increase in unit weight of the dewatered deposits multiplied by the dewatered thickness. Use of the assumed unit weights indicates that the effective stress below the lowered water table is increased 0.6 foot for each foot of water-table lowering.

CONFINED (ARTESIAN) CONDITIONS

Under simple hydrostatic conditions in a confined aquifer system (fig. 4A), the hydraulic heads above and below the confining layer stand at the same altitude. Therefore no hydraulic gradient exists across the confining layer and, consequently, no seepage occurs through the confining layer. The effective stresses

under these hydrostatic conditions are the same as in the unconfined case, that is, equal to the gravitational stress of the overlying deposits.

If both the water table and the piezometric head in the confined aquifer are 200 feet below land surface (fig. 4A), the two components of effective stress exerted downward on the grains, due to the weight of deposits above the water table (vector p_s') and due to the buoyant weight of deposits below the water table (vector p_b') are 320 feet and 200 feet, respectively, on the top of the confining layer; 320 feet and 300 feet on the top of the confined aquifer system, and 320 feet and 500 feet on the top of the aquitard at a depth of 700 feet.

If the artesian pressure in a confined aquifer system, under the hydrostatic condition of figure 4A, is drawn down below an unchanging water table by pumping (fig. 4B), gravitational stresses remain the same; however, the downward hydraulic gradient developed across the confining layer (aquiclude) induces downward seepage. The loss of hydraulic head through the aquiclude represents the energy expended in moving the water through the pore passages. As head is dissipated by viscous flow through the confining layer, the energy in the moving water is transferred by viscous drag to the granular skeleton of the deposit. The force corresponding to this energy transfer is exerted in the direction of flow and is algebraically additive with the gravitational stresses that tend to compact the deposits.

As commonly used, the term "seepage force" refers to the energy transfer that occurs within the body through which the seepage takes place. As defined by Scott (1963, p. 96, equation 4-20), the seepage force, f , exerted on the grains can be represented by the expression

$$f = h \cdot \gamma_w \cdot A,$$

in which h is equal to the head loss across an incremental thickness, and A is equal to the cross sectional area.

Within an aquiclude that is compacting under the stress of downward seepage (fig. 4B), the hydraulic gradient, and therefore the seepage force per unit thickness, varies from a minimum at the upper surface to a maximum at the lower surface until steady-state conditions are reached. The loading effect of the incremental seepage force developed through each unit volume vertically through the aquiclude is additive downward. Thus, the net effect of these incremental units of seepage force at any point in the structure of the aquiclude is equal to the summation of the incremental forces above that point. Although the distribution of these forces within the aquiclude may be indeterminate, the total seepage force accumulated through the aquiclude is equal to the total head dif-

ferential across it. Thus, at the base of the aquiclude, the total seepage force, F , may be expressed as

$$F = H \cdot \gamma_w \cdot A,$$

and the seepage force per unit area, J , hereafter referred to as the seepage stress, is

$$J = \frac{F}{A} = H \cdot \gamma_w.$$

This seepage stress is transmitted downward through the intergranular structure of the aquifer system below the aquiclude. If stress or pressure is expressed as equivalent head of water instead of force per unit area, the specific weight of water, γ_w , which serves to convert head into pressure units, is not required, and the expression is simply $J = H$.

In the assumed ground-water systems of figure 4B, straight line $A-B-C-D$ represents the hydrostatic pressure (neutral pressure) gradient that existed before pumping began. The deposits are adjusted to the overburden load which equals the actual weight of the deposits above the water table plus the buoyant weight of the submerged deposits. After pumping has lowered the head in the permeable members of the confined aquifer system to 300 feet, the effective stresses shown by vectors in figure 4B would prevail at the depths represented by the arrow points. The length $C-F$ represents the 100 feet of head decline, and $F-G$ represents the reduced fluid pressure in the confined aquifer system. With a 100-foot head loss through the confining layer, a 100-foot seepage stress is immediately developed across the confining layer and transmitted through grain-to-grain contacts to the underlying aquifer system. Even though normal hydrostatic pressure gradients exist in both the unconfined (line $A-B$) and the confined (line $F-G$) aquifers, hydraulic gradients and transient flow conditions within the confining aquiclude gradually change until a steady hydrodynamic gradient (line $B-F$) is established between the heads at the upper and lower surfaces of the aquiclude. The initial head distribution through the confining aquiclude is approximated by lines $B-C$ and $C-F$, and the transient distribution of head in the confining layer at some intermediate time before steady flow is established is approximated by the curved line $B-E-F$.

As shown in figure 4B, the length, s , indicates the magnitude of the seepage stress at a particular depth in the aquiclude, and the length, u , indicates the excess over steady-state head (Taylor, 1948, p. 221, 236) that remains in the interstitial voids at that depth. Also, the area $BCFEB$ is proportional to the part of the ultimate compaction that has occurred and area $BEFB$ to the part of the ultimate compaction that has

not taken place. Domenico and Mifflin (1965, p. 571) referred to the combined area $BCFEB$ as the "effective-pressure area". As adjustment continues, excess pore pressures are reduced and compaction occurs within the aquiclude. After equilibrium conditions have been established, head distribution through the aquiclude follows straight line $B-F$ (assuming uniform permeability), and uniform flow conditions and a constant seepage gradient prevail through the aquiclude. Under these conditions, 1 foot of head is lost (transferred to the grains) through each foot of thickness of the aquiclude. The accumulated downward seepage thrust exerted on the grains by the moving water thus varies linearly from 0 at the upper surface of the aquiclude to 100 feet at its lower surface. The total seepage stress, equal to the head differential across the aquiclude, is exerted on the granular structure of all underlying beds in the aquifer system. The time involved in reaching complete adjustment is dependent chiefly on the thickness, compressibility, and permeability of the confining layer.

At all times after the head in the confined aquifer system is lowered to 300 feet, the magnitude of the seepage stress (increased effective stress) exerted downward on all beds within the confined system (below the aquiclude) is 100 feet (distance $C-F$, fig. 4B). In the more permeable, less compactible aquifers of the confined systems, the increase in effective stress occurs rapidly. Within the aquitards, however, no change in effective stress can occur until water escapes from these slow-draining, compacting beds. The reduction in head in the permeable aquifers creates a two-directional hydraulic gradient outward from the center of the aquitard and thereby induces drainage from and compaction of the aquitard. The drainage is two-directional and excess pore pressures are transient (curved line $H-I-J$, fig. 4B) until equilibrium conditions (straight line $H-J$) are established. Although upward and downward seepage stresses occur within the aquitard during this adjustment, they have no net external effect on the rest of the aquifer system.

The open arrows ($\Delta p'$) at two depths in the hydrologic section of figure 4B are vectors indicating the change in effective stress from the hydrostatic conditions of figure 4A. Thus, the net effect of lowering the piezometric head in the confined aquifer system 100 feet below the water table is to create a seepage stress through the confining layer and, ultimately, to increase the effective stress in all beds in the confined aquifer system by 100 feet. No change in effective stress occurs in the unconfined aquifer overlying the confining layer. Although a small part of the resulting deformation of the confined aquifer system is elastic

and reversible, a large part represents an irreversible rearrangement of fine particles in the clayey sediments to a more dense state.

Figure 4C represents a dual water-level lowering, with both the water table and piezometric surface lowered 100 feet below the level in figure 4A. A normal hydrostatic gradient of pressure with depth exists in the aquifers and is ultimately achieved in both the aquiclude and the aquitard. However, effective stresses throughout the section are considerably different from the conditions of either figure 4A or figure 4B. Loss of buoyant support in the dewatered zone causes the effective stress changes shown in figure 4C. The net change in stress from conditions of figure 4A is shown at three depths by the open arrow vectors. Although the effective stress throughout the section is increased from conditions of figure 4A by the loss of buoyant support, the stresses in the confined aquifer system are less than those of figure 4B because there is no seepage stress component. It is apparent that if the head in the confined aquifer system had been drawn down to the condition of figure 4B before the hydrostatic conditions of figure 4C were established, rebound would tend to occur throughout the confined aquifer system as the water table was drawn down.

If the piezometric surface remains constant and the water table is raised 100 feet above the conditions of figure 4A, the effective stresses shown by vectors in figure 4D are produced. The raised water table reduces the gravitational stress below the raised level by providing buoyant support of the grains in the zone of water-table rise; it also creates a downward seepage stress across the confining layer. The net change from the conditions in figure 4A is to increase effective stress throughout the confined aquifer system. It is noteworthy that rebound would tend to occur in the unconfined aquifer when the conditions of figure 4A are changed to the conditions of figure 4D, whereas a 40-foot increase in stress would occur in the underlying confined system.

If the water table is raised 100 feet above the hydrostatic conditions of figure 4A, and the piezometric surface is lowered 100 feet, the effective stresses shown in figure 4E will prevail. Increase in buoyant support causes reduced gravitational stress in the unconfined aquifer, as it did under the conditions of figure 4D. The 200 feet of head differential across the confining layer represents a seepage stress of 200 feet exerted downward on the deposits in the confined aquifer system. Under these conditions, the change in effective stress in the confined aquifer system, due to the combined changes in gravitational and seepage stress from the hydrostatic condition of figure 4A, is 140 feet,

which is considerably greater than for conditions illustrated in figures 4B, C, and D.

Figure 4F shows the effective stresses under the sixth general condition where the water table is lowered 100 feet below the piezometric surface of figure 4A. Here the effective stress in the unconfined aquifer is increased 60 feet due to the loss of buoyant support in the zone of lowered water table. Because of upward seepage stresses that are developed through the aquiclude, however, the effective stress throughout the confined aquifer system is reduced by 40 feet from the hydrostatic condition of figure 4A.

SUMMARY

The following generalizations are made with respect to analysis of effective stress:

1. A ground-water reservoir may act as an unconfined aquifer system if no significant vertical head differentials (seepage stresses) occur within it under ordinary pumping stress. It includes one or more confined (artesian) systems if vertical head differentials within the reservoir are significant.
2. If the porosity of an unconfined aquifer is 40 percent, the effective stress in the aquifer would increase 0.6 foot for each foot of water-table decline and would decrease 0.6 foot for each foot of water-table rise. For larger or smaller values of porosity the values of effective-stress change would be somewhat different. The change in effective stress equals the change in buoyant support of the deposits in the interval of water-table change and is independent of the specific gravity of the grains.
3. If a confined aquifer is overlain by an unconfined aquifer, the effective stress is affected by changes in both the water table and the piezometric surface. If the porosity is 40 percent, a water-table rise of 1 foot would cause a 0.4-foot increase in effective stress whereas a decline of 1 foot in the piezometric surface would cause a 1-foot increase in effective stress.

REFERENCES

- Domenico, P. A., and Mifflin, M. D., 1965, Water from low-permeability sediments and land subsidence: *Am. Geophys. Union, Water Resources Research*, v. 1, no. 4, p. 563-576.
- Lofgren, Ben E., 1961, Measurement of compaction of aquifer systems in areas of land subsidence: *Art. 24 in U.S. Geol. Survey Prof. Paper 424-B*, p. B49-B52.
- Scott, R. F., 1963, *Principles of soil mechanics*: Palo Alto, Calif., Addison-Wesley Pub. Co., Inc., 550 p.
- Taylor, D. W., 1948, *Fundamentals of soil mechanics*: New York, John Wiley and Sons, 700 p.
- Terzaghi, Karl, and Peck, R. B., 1948, *Soil mechanics in engineering practice*: New York, John Wiley and Sons, 566 p.