

FIELD VERIFICATION OF A ONE-DIMENSIONAL MATHEMATICAL MODEL FOR TRANSIENT
COMPACTION AND EXPANSION OF A CONFINED AQUIFER SYSTEM

Donald C. Helm¹

Land subsidence due to ground-water withdrawal from a confined aquifer system is an expression at land surface of net compaction (vertical consolidation) at depth of compressible layers within the system. Water-level fluctuations within coarse-grained aquifers produce stress changes on the upper and lower boundaries of slow-draining aquitards (compressible fine-grained interbeds). Indirectly, irregular and cyclic pumping drawdowns generate complex loading and unloading stress patterns in the aquifer system, causing compaction and expansion of the aquitards. Within a confined system, any lag in compactive response to such a load increase is usually ascribed to slow vertical drainage from the aquitard(s). After water levels in coarse-grained material rise during a period of water-level recovery, it has been observed during the next cycle of declining water levels that nonrecoverable compaction is reinitiated before water levels reach their prior lowest level.

Based on a modification of Terzaghi's consolidation theory, a one-dimensional mathematical model has been developed and discussed in detail by Helm (3, 4). The model computes transient aquitard compaction and expansion due to arbitrarily specified changes in effective stress within an adjacent aquifer. Conceptually, the model attributes time-lag of matrix response to vertical flow of fluid mass from one idealized material (such as one or more aquitards) to another (such as adjacent aquifer material). The present model has been applied to various field conditions ranging from a short-term test where 4.8×10^{-4} meters of recoverable compaction (and subsequent expansion) of an overlying confining bed was measured during a 4-week aquifer test (including drawdown and recovery) in Florida to a long-term test where nearly 4 meters of nonrecoverable subsidence occurred in San Jose, Calif., during 50 years of pumping overdraft.

The present model differs fundamentally from a viscoelastic type of model (1, 2) that makes the following simplifying assumptions: 1) a confined aquifer system is represented by a single undifferentiated viscoelastic material, 2) time-lag of matrix response to observed stress change within such an undifferentiated aquifer system is attributed to a viscous element of the rheological model, 3) vertical components of flow are ignored, and 4) vertical variation in hydraulic head and pre-consolidation pressure is ignored.

The present model computes not only transient compaction and expansion at a site of interest (Figure 1) but also the transient vertical distribution of past maximum effective stress (preconsolidation pressure) (Figure 2). If adequate background data are available, the model can be used not only to predict transient compaction and expansion at a site due to arbitrarily specified loading and unloading patterns, but also

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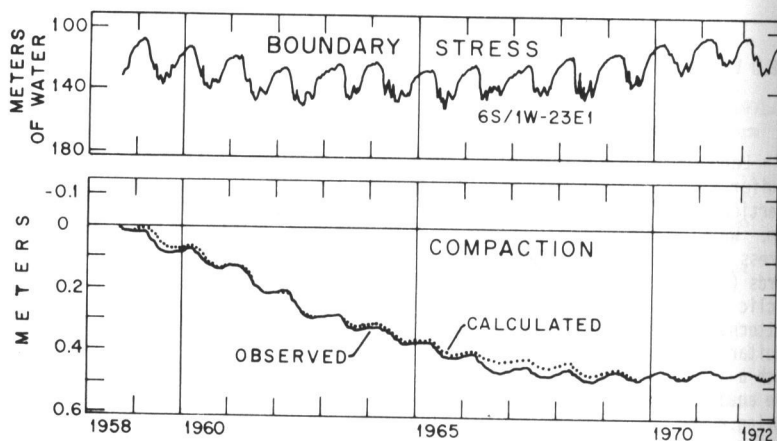


Figure 1.--Simulation of compaction based on water-level and com-
data for well 6S/1W-23E1 (1958-72).

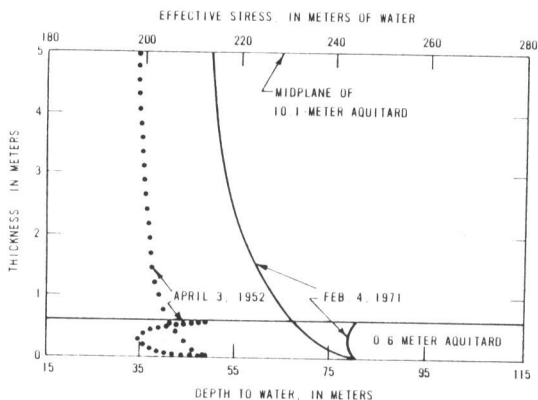


Figure 2.--Combined plot showing distribution of calculated precon-
solidation pressure within two idealized aquitards at the -16N3
field site on April 3, 1952 (dotted lines), and on February 4,
1971 (solid lines). Longer segments of curves represent distribu-
tion in lower half of 10.1-meter aquitard; shorter segments
represent distribution in the 0.6-meter aquitard.

to approximate quantitatively a critical range of water levels. Some-
where within this critical range the nonrecoverable component of com-
paction is predicted to cease when water levels rise and to be reinit-
iated during subsequent water-level declines.

For more plots similar to Figure 1 that simulate observed compac-
tion and expansion at a number of sites in California, see Helm (5).

The boundary stress curve of Figure 1 has been calculated directly from observed water-level data by adding an average effective weight of the overburden (3, 4). The emphasis of the present paper will be threefold: First, the trial-and-error method used by the author to find vertical hydraulic conductivity, K' , and nonrecoverable vertical specific storage (S'_{skv}), will be described. Second, the values of K' and S'_{skv} estimated by this method based on 15 sets of field data in California will be listed. Finally, based on these values important predictions of critical stress values and ultimate compaction values will be presented.

The first step in searching for appropriate parameter values is to select arbitrarily a value for hydraulic conductivity, K' , and nonrecoverable specific storage, S'_{skv} , for an initial computer run. Let us label this initial set ($K'(a)$, $S'_{skv}(a)$). A computed compaction curve will be generated by the model in response to known or otherwise specified changes in boundary stress. If, for example, one finds that the calculated compaction curve gives a value for net cumulative compaction between two selected dates (t_1 and t_2) larger than the observed values, one can decrease K' or S'_{skv} or both for the next trial computer run(s).

For a few hypothetical subsequent runs let us keep $K' (= K'(a))$ constant and decrease only S'_{skv} . For an extreme case, zero-valued S'_{skv} (representing incompressible intergranular material) will obviously give a calculated compaction curve smaller than the observed non-zero compaction. It is possible to find a value of S'_{skv} (namely, $S'_{skv}(b)$ in Figure 3) for our initial arbitrary $K'(a)$ that will match the net

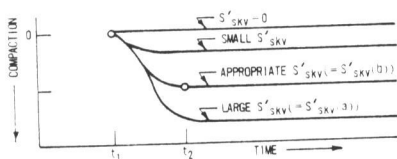


Figure 3.--Hypothetical computer runs for a large value of $K' (= K'(a))$.

observed compaction between t_1 and t_2 . The two circles in Figures 3, 4, and 6 represent hypothetical observed values of compaction at $t = t_1$ and t_2 .

Let us now keep the hypothetical initial arbitrary $S'_{skv}(=S'_{skv}(a))$ constant and decrease K' (Figure 4) on subsequent computer runs. Eventually we shall have two possible pairs of parameters that adequately

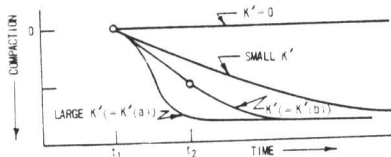


Figure 4.--Hypothetical computer runs for a large value of $S'_{skv}(=S'_{skv}(a))$.

account for observed net compaction between t_1 and t_2 , namely $(K'(a), S'_{skv}(b))$ from Figure 3 and $(K'(b), S'_{skv}(a))$ from Figure 4. These two pairs of values represent two points on a "zero-error" curve (Figure 5) between t_1 and t_2 .

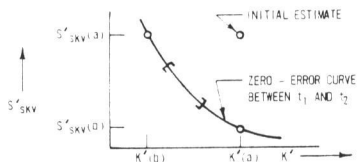


Figure 5.-- Hypothetical zero-error curve.

When more observed time-compaction data are available than merely two values (circles in Figures 3 and 4), we can by trial-and-error find many pairs of coefficients (bracketed segment of the "zero-error" curve in Figure 5) that give a reasonable approximation of the slope of an observed time-compaction curve between t_1 and t_2 (Figure 6). These

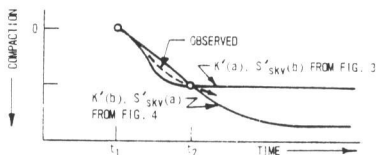


Figure 6.--Hypothetical observed slope.

values may be found to fall within a small range. Hence, the coefficient pairs in Figure 5 represented by the bracketed interval on the so-called "zero-error" line are found, not assumed. What is assumed is the appropriateness of Terzaghi's equation. In other words, the values in Figure 5 are actually values for mathematical coefficients in an equation. If the mathematical equation does indeed represent a physical process, then the mathematical coefficients represent corresponding physical parameters. What is assumed is a correspondence between a selected mathematical equation and an observed physical event.

If field data are not related in a cause-and-effect relation, no equation (theory) can be tested and no meaningful "zero-error" coefficient values can be found. Helm (5) has discussed and illustrated the application of the "zero-error" curve method (Figure 5) to three sets of actual field data from the Santa Clara Valley, California.

Columns 8 and 9 in Table 1 give parameter values for K' and S'_{skv} (estimated by the previously described method) for eight sets of field data in the Santa Clara Valley, Calif., and seven sets of field data in the San Joaquin Valley, Calif. Column 1 indicates the location number of observation wells whose water-level fluctuations were used as known changes in boundary stress on the idealized aquitards. A location map of the specified observation wells in Santa Clara Valley can be found in

Table 1.--Values of parameters used for simulating observed compaction at selected sites in Santa Clara and San Joaquin Valleys, California

Site of water-level data	Period of record (years)		Estimated thickness (meters)			Estimated parameter values			
	Water-level data	Compaction data	b' 0 sum	Total b' 0 total	Weighted average b' 0 equiv	Thickest aquitard b' 0 max	K' meter x 10 ⁻⁴ year	S' skv 1 meter x 10 ⁻⁴	S' ske 1 meter x 10 ⁻⁶
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
SANTA CLARA VALLEY									
6S/1W-23E1	1958-72	1958-72	57.6	68.6	4.6	-	1.31	4.6	24.6
-24C3	1960-72	1960-72	77.7	111.3	5.5	10.4	.91	10.8	13.1
-24C7	1960-72	1960-72	128.6	137.2	16.2	25.9	13.72	11.0	8.5
6S/2W-25C1	1960-72	1960-72	77.7	111.3	5.5	10.4	.84	10.7	52.5
-25C1	1921-74	1932-74	77.7	111.3	5.5	10.4	.81	10.8	-
7S/1E-7R1	1915-74	1916-74	145.4	243.8	5.5	16.2	2.07	7.4	-
-902	1958-72	{1960-63} {1965-72}	78.6	134.1	5.5	16.2	.79	13.1	26.2
-16C5	1958-72	1960-72	145.4	243.8	5.5	16.2	3.66	7.5	7.2
SAN JOAQUIN VALLEY									
11N/21W-3B1	1963-74	1963-74	111.9	204.2	5.8	19.8	.91	8.2	13.1
14S/13E-11D3,6	1961-74	1961-74	83.5	176.2	5.2	10.1	2.35	14.1	23.0
16S/15E-34N4	{1960-63} {1964-74}	1958-74	267.0	395.3	5.8	14.3	1.58	7.9	7.2
18S/19E-20P2	1967-74	1967-74	46.9	127.1	9.1	10.1	2.13	22.0	24.6
19S/16E-23P2	1959-74	1960-74	403.6	597.4	5.8	14.3	.06	9.8	6.6
20S/18E-11O1	1958-74	1964-74	118.3	189.0	6.7	25.6	.37	4.6	13.1
23S/25E-16N3	1952-71	1959-74	84.7	123.4	4.9	10.1	9.14	7.5	15.1

Helm (5). Location maps of the specified sites in the San Joaquin Valley appear in Poland and others (8, Figure 50). Column 2 lists the period of water-level data used and column 3 gives the period of measured compaction used. It is these compaction data measured in the field by extensometers and described by Lofgren (6) that are to be matched by calculated compaction (Figure 1). In column 4, $b'_0 \text{ sum}$ is the fine-grained part of the total thickness interval ($b'_0 \text{ total}$ of column 5) that has experienced the stress change and whose gross cumulative compaction and expansion has been measured by extensometers. Column 6 gives the weighted-average thickness, $b'_0 \text{ equiv}$, of aquitards in contrast to the thickest aquitard ($b'_0 \text{ max}$ of column 7) within the total interval ($b'_0 \text{ total}$ of column 5) of interest. The method of calculating $b'_0 \text{ equiv}$ is discussed in (3, 4, 9). Values in columns 4 through 7 are estimated directly from electric or other down-hole logs. The recoverable component of specific storage, namely S'_{skv} of column 10, is estimated directly from a plot of field data according to a method described by Riley (9). It is interesting to note that the model values of S'_{skv} listed in column 9 and found by the previously described trial-and-error method fall within an unexpectedly narrow range.

Table 2 shows some predictions based on the simulation model. Columns 2 and 3 are observed water levels for the coarse-grained material. Column 2 indicates that water levels in the early 1970's had risen from their earlier historical lows of the mid-1960's (column 3). Column 4 represents calculated preconsolidation pressure (Figure 2) at the end of the appropriate periods of record (column 2 of Table 1). The "high" water-level value in column 4 of Table 2 is a very rough approximation for the end-date preconsolidation pressure at the midplane of the thickest aquitard ($b'_0 \text{ max}$ in column 7 of Table 1). It is intended to suggest a possible upper limit for a critical depth to water. The "low" water-level value in column 4 of Table 2 is a much firmer number. It is the calculated end-date preconsolidation pressure at the midplane of an "average" aquitard ($b'_0 \text{ equiv}$ in column 6 of Table 1) and represents a lower limit for the transient critical depth to water mentioned above. For water levels below such a critical depth some nonrecoverable subsidence will occur. For water levels above this critical depth, none will occur. The critical depth to water is predicted to lie somewhere between the two values in column 4 of Table 2. Table 2 can be used along with Figure 2 to illustrate that at site 23S/25E-16N3 (column 1) the $b'_0 \text{ max}$ aquitard with a thickness of 10.1 meters (column 7, Table 1) had in 1971 (column 2, Table 1) a calculated preconsolidation stress at the midplane that is equivalent to a depth to water of 50 meters (column 4, Table 2). Column 4 of Table 2 also indicates that at site 23S/25E-16N3 the representative aquitard with a modeled thickness of 4.9 meters (column 6, Table 1) had in 1971 a calculated preconsolidation stress at its midplane that is equivalent to a depth to water of 67.1 meters. Hence the 1971 critical depth to water at this site should lie somewhere between 50. and 67.1 meters. Column 5 of Table 2 lists the residual nonrecoverable compaction at each site that will occur if water levels are returned to their past maximum (column 3, Table 2) and held at that level indefinitely. Column 6 indicates the calculated ultimate specific compaction as defined by Poland and others (7). It simply equals $b'_0 \text{ sum} \times S'_{skv}$ and is equivalent to a nonrecoverable storage coefficient S'_{kv} . For example, if water levels within the specified thickness interval of 123.4 meters at 23S/25E-16N3 were lowered 10 meters

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Table 2.--Quantitative predictions based on simulated compaction at selected sites in Santa Clara and San Joaquin Valleys, California

Site of water-level data	Depth to water (meters)			Residual nonrecoverable compaction (meters)	Ultimate specific compaction, S'_{kv} (dimensionless)	Time constant τ (years)
	Early 1970's	Max. past mid-1960's	Range to "trigger" subsidence High + Low			
(1)	(2)	(3)	(4)	(5)	(6)	(7)
SANTA CLARA VALLEY						
65/1W-23E1	30.	57.3	- + 30.5	0.52	0.0265	18.
-24C3	30.	50.3	3.7 + 25.6	1.58	.0842	89.
-24C7	30.	43.0	23. + 30.5	1.25	.141	52.
65/2W-25C1	30.	50.9	0. + 23.8	1.71	.0829	96.
-25C1	30.	50.9	0. + 23.5	1.74	.0842	100.
75/1E-7R1	30.	56.1	1.2 + 37.5	1.52	.107	27.
-902	40.	62.5	0. + 30.5	2.53	.155	125.
-16C5	45.	82.6	28. + 64.0	1.65	.110	13.
SAN JOAQUIN VALLEY						
11W/21W-3B1	120.	164.6	10. + 119.5	3.20	0.092	75.
14S/13E-11D3.6	110.	155.1	106. + 129.8	2.19	.118	40.
16S/15E-34N4	125.	188.4	115. + 158.5	4.57	.210	42.
18S/19E-20P2	40.	70.1	47. + 49.7	1.89	.103	215.
19S/16E-23P2	155.	185.3	110. + 155.1	9.75	.397	1350.
20S/18E-11Q1	90.	150.9	0. + 73.2	2.96	.054	140.
23S/25E-16N3	45.	83.2	50. + 67.1	.85	.064	5.

below their past maximum depth of 83.2 meters and were held there long enough, 0.64 meters of nonrecoverable compaction is predicted to occur ultimately in addition to 0.85 meters of residual nonrecoverable compaction for a total of 1.49 meters of nonrecoverable compaction at that site. Column 7 gives the time constant τ which equals $S'_{skv}(b'_{0 \text{ equiv}}/2)^2/K'$. The calculated value of τ suggests the length of time required at each site for the slow-draining aquitards to reach 93 percent of their ultimate nonrecoverable compaction (column 6) if water levels in the aquifers were instantaneously lowered a specified amount and then held constant. The τ value listed for each site in Table 2 is a gross, but representative approximation. The value of for any bed depends on the square of the thickness of the individual bed itself. The calculated value of τ in Table 2, however, depends on the square of a weighted-average thickness, $b'_{0 \text{ equiv}}$. Because τ also depends on field-determined average values of K' and S'_{skv} for the entire system, it may be considered a reasonable value for each site.

Appendix 1.—References

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GROUNDW

David

SUMMAR

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*Prof
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